Título: Structural and complexity studies in inversions and colouring

heuristics of (oriented) graphs

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Resumo:

This thesis is divided in two parts, where the first one concerns the inversion number of oriented graphs. Let D be an oriented graph. The inversion of a set X of vertices in D consists in reversing the direction of all arcs with both extremities in X. The inversion number of D, denoted by inv(D), is the minimum number of inversions needed to make D acyclic. We studied the relation between the inversion number and other parameters related to problems of making a digraph acyclic such as CYCLE ARC TRANSVERSAL (or F EEDBACK ARC SET) and CYCLE TRANSVERSAL (or FEEDBACK VERTEX SET). The cycle transversal (resp. cycle arc-transversal) number of a digraph D, denoted by $\tau(D)$ (resp. $\tau'(D)$), is the size of a minimum set of vertices (resp. arcs) whose removal makes D acyclic. The cycle packing number of D, denoted by $v(D)$, is the maximum size of a disjoint set of cycles of D. We show that $inv(D) \leq \tau'(D)$, $inv(D) \leq 2\tau(D)$ and there exists a function g such that $inv(D) \leq g(v(D))$, where v(D). We conjecture that for any two oriented graphs L and R, $inv(L \rightarrow R) = inv(L) + inv(R)$ where $L \rightarrow R$ is the dijoin of L and R. This would imply that the first two inequalities are tight. We prove this conjecture when $inv(L) \leq 1$ and $inv(R) \leq 2$ and when $inv(L) = inv(R) = 2$ and L and R are strongly connected. We also show that the function g of the third inequality satisfies $g(1) \leq 4$. We then consider the complexity of deciding whether $inv(D) \leq k$ for a given oriented graph D. We show that it is NP-complete for $k = 1$, which together with the above conjecture would imply that it is NP-complete for every k. This contrasts with a result of Belkhechine et al. (BELKHECHINE et al., 2010) which states that deciding whether $inv(T) \leq k$ for a given tournament T is polynomial-time solvable. The second part of this work is about b-greedy colourings and z-colourings. A b-greedy colouring is a colouring which is both a b-colouring and a greedy colouring. A z-colouring is a b-greedy colouring such that a b-vertex of the largest colour is adjacent to a b-vertex of every other colour. The b-Grundy number (resp. z-number) of a graph is the maximum number of colours in a b-greedy colouring (resp. z-colouring) of it. In this part, we study those two parameters. We show that they are not monotone and that they can be arbitrarily smaller than the minimum of the Grundy number and the b-chromatic number. We prove that it is NP-hard to compute each of those parameters. However, we describe a polynomial-time algorithm that decides whether a given k-regular graph has b-Grundy number (resp. z-number) equal to $k + 1$. We also prove that, except for the Petersen graph, every cubic graph with no induced 4-cycle has b-Grundy number and z-number exactly 4.

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